Thermal Response Factors for Vertical Ground Heat Exchangers

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The objective of this article is to demystify g-functions, which are used in many software tools as thermal response factors to model the thermal behavior of vertical ground heat exchangers (VGHEs). This article follows a companion article[1] that described the use of g-functions for sizing VGHEs. The companion article includes a spreadsheet tool, GHXSizing, that can evaluate g-functions. This article explains the methods to evaluate g-functions and provides several examples of their use.

Introduction to g-functions

Various borehole sizing models ranging from simple three-pulse methods to hourly methods are presented in a previous article.[1] This article introduces g-functions, which are used in some of these methods.

Thermal response factors for VGHEs can be obtained using g-functions, which were first introduced in Sweden by Eskilson in his seminal work.[2] As shown in *Equation 1* and *Figure 1*, g-functions, $g(t)$, are used to predict the borehole wall temperature, T_b, at time t for a given heat extraction rate per unit length, q_b, ground thermal conductivity, k_g, and undisturbed ground temperature, T_g.

$$T_b(t) - T_g = \frac{q_b}{2\pi k_g} g(t) \tag{1}$$

Equation 1 does not account for transient heat transfer within the borehole. Short-term g-functions (based on a slightly different formulation that involves the mean fluid temperature, T_m, steady-state borehole thermal resistance, R_b, and heat transfer rate to the fluid, q_f) can be used to account for these short-term effects.[3,4,5] In this article, short-term effects are not considered, and it is assumed that q_b is equal to q_f.

Figure 2 shows a graphical representation of g-function curves for a 3×1 borehole field. In this graph, g-function curves are plotted as a function of $\ln(t/t_s)$ for $r_b/H = 0.0005$, $D/H = 0.01$, and six values of the B/H ratio (the curve for $B/H = \infty$ corresponds to the g-function of a single borehole). In these terms, H is the borehole length, t is the time, t_s is the characteristic time ($= H^2/9\alpha$, where α is the ground thermal diffusivity), r_b is the borehole radius, B is the separation

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distance between boreholes, and D is the distance from the ground surface to the top of the boreholes. Much like the Moody diagram used to evaluate pipe friction, which gives f, the friction factor, as a function of the nondimensional Reynolds number and roughness ratios, g-function graphs provide the value of g as a function of $\ln(t/t_s)$ and three nondimensional parameters, r_b/H, B/H and D/H. This last parameter has a negligible impact except for short boreholes, and it will be neglected here.

It is interesting to note that all g-function curves in *Figure 2* merge in the region where $\ln(t/t_s) < -5$ (this value changes slightly with bore field geometry). In this region, thermal interaction among boreholes has not yet started, and all boreholes behave as single boreholes. For $\ln(t/t_s) > -5$, the curves start to split as boreholes begin to interact with each other. Curves for small values of the B/H ratio are the first to diverge because the boreholes are closer to each other for these ratios. For $\ln(t/t_s) > 2.5$, the curves have reached a plateau, indicating that g-functions are constant and the bore field has reached a "steady-state" condition.

Table 1 provides a first example of the use of g-functions for two different geometries. Parameters have been selected so that both geometries have identical nondimensional parameters: $B/H = 0.05$, $D/H = 0.01$, $r_b/H = 0.0005$ and $\ln(t/t_s) = -1.1$ (*Equation 1*). *Figure 2* yields a g-function of 8.66 for these values (this g-function value is the same for both geometries since they have the same nondimensional parameters). Thus, if q_b is the same in both cases, then the borehole wall temperature (T_b) will also be identical. However, it is important to note that these borehole wall temperatures are obtained after 10 years and 22.8 years of operation, respectively, since the geometries have different characteristic times (t_s) even though they have the same $\ln(t/t_s)$.

g-functions in *Figure 1* are valid only for $r_b/H = 0.0005$. For different r_b/H ratios, the correction factor suggested by Eskilson[2] should be used:

$$g = g^o - \ln\left(\left(\frac{r_b}{H}\right) / \left(\frac{r_b}{H}\right)^o\right) \qquad (2)$$

where the superscript ^{o} refers to the original values. For example, if $r_b/H = 0.00075$, the new g-function will be

FIGURE 1 Parameters involved in g-functions.

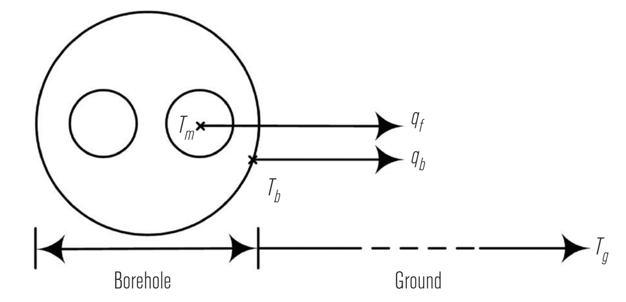


FIGURE 2 g-function curves for a 3 × 1 borehole field with six different borehole spacings for D/H = 0.01 and r_b/H = 0.0005.

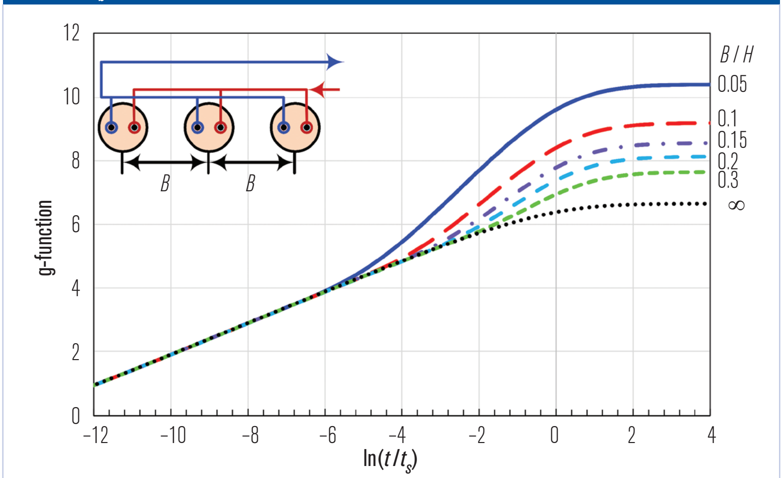


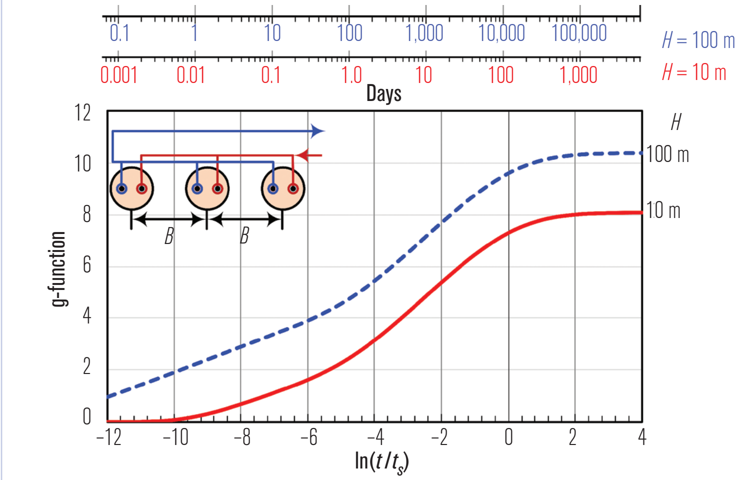
TABLE 1 Parameters for the two geometries used in the examples.

GEOMETRY	A	B
Size of Bore Field	3 × 1	
Ground Thermal Diffusivity, α (m²/day)	0.1	
Ground Thermal Conductivity, k_g (W/m·K)	1.0	
Ground Temperature, T_g (°C)	0	
Borehole Radius, r_b (m)	0.05	0.075
Distance from Ground Surface to Top of Borehole, D (m)	1.0	1.5
Borehole Spacing, B (m)	5.0	7.5
Borehole Length, H (m)	100	150
r_b/H (–)	0.0005	0.0005
D/H (–)	0.01	0.01
B/H (–)	0.05	0.05
Characteristic Time, t_s (yr)	30.4	68.5
Time, t (yr)	10	22.8
$\ln(t/t_s)$ (–)	−1.1	
g (–)	8.66	

$8.25\ (= 8.66 - \ln\left(\frac{0.00075}{0.0005}\right))$.

Figure 3 shows the impact of borehole length on g-functions for a 3 × 1 bore field. In this figure, B/H = 0.05 and D/H = 0.01 for both curves. The borehole radius is also the same for both (0.05 m [2 in.]), but values of r_b/H are different (0.005 and 0.0005) because the borehole lengths are different. The top scale provides the dimensional time for both lengths on a logarithmic scale. For H = 10 m (32.8 ft), steady-state condition is reached after ~1,000 days, while reaching this condition may take significantly more time for H = 100 m (328 ft). A g-function of ~4 is observed after ~5 days for H = 10 m (32.8 ft), while for H = 100 m (328 ft), it takes ~30 days to reach this g-function value.

FIGURE 3 Effect of borehole length on g-functions for r_b = 0.05 m (2 in.), B/H = 0.05, D/H = 0.01.



Thus, for example, if q_b is set to 6.28 W/m (6.53 Btu/h·ft) (giving a ratio of $q_b/2\pi k_g$ of one in SI units and 1.8 in IP units) and T_g = 0°C (32°F), then the borehole wall temperature (*Equation 1*), T_b, will reach a value of 4°C (39.2°F = 32°F + 1.8 × 4) after 5 and 30 days, respectively. However, it should be kept in mind that q_b is the heat transfer rate per unit length, and thus the 10 m (32.8 ft) long borehole will transfer 10 times less heat than the 100 m (328 ft) borehole.

Evaluation of g-functions

Most software tools have databases of precalculated g-functions for several hundred standard bore field configurations, and designers typically do not need to evaluate their own g-functions. However, some insight about two-dimensional borehole heat transfer and borehole thermal interaction can be gained by examining their evaluation. Readers are referred to the works of Eskilson,[2] Cimmino,[6] and Cimmino and Bernier[7] for more complete details. Furthermore, some open-source scripts can generate g-functions.[8,9]

The following example consists of determining a particular point on the g-function curve for a 3 × 1 bore field corresponding to geometry A in *Table 1*. The objective of the calculations for this example is to obtain the borehole wall temperature, T_b, for a given heat transfer rate per unit length, q_b, after 10 years of operation for a 100 m (328 ft) borehole in order to determine the g-function (using *Equation 1*).

g-functions were initially evaluated by Eskilson[2] with a 2D numerical model around a single borehole, and spatial superposition was used to account for borehole-to-borehole thermal interactions.

The solution used here is based on the work of Cimmino and Bernier[8] who have shown that the finite line source (FLS) analytical solution can be used instead of a numerical model, yielding a significant reduction in calculation time. The FLS solution treats the boreholes as finite lines subjected to a uniform heat transfer rate per unit length, q_b, and it applies a mirror image above ground to account for the ground surface.[2,10] The FLS solution assumes that heat transfer in the ground is by pure conduction only. Because of the nondimensional nature of g-functions, they can be determined using any values of q_b and T_g. In this example, values of q_b are set at 6.28 W/m (6.53 Btu/h·ft) and T_g = 0°C (32°F) are for convenience, as the calculated value of T_b is then simply equal to the g-function in SI units (and to 32°F to 1.8 times the g-function in IP units) (*Equation 1*).

Figure 4 shows the 2D ground temperature profiles (radial and axial) obtained using the FLS solution at three radial distances corresponding to r_b = 0.05 m (2 in.), B = 5 m (16.4 ft) (distance between two adjacent boreholes) and $2B$ = 10 m (32.8 ft) (distance between the two outer boreholes) after 10 years of operation, with q_b = 6.28 W/m (6.53 Btu/h·ft) and T_g = 0°C (32°F). The two-dimensional nature of the temperature profile can be seen, with each profile having an ellipse-like shape.

For example, the ground temperature for B = 5 m (16.4 ft) increases rapidly going downward from the surface and reaches a maximum value of ~1.7°C (~35°F) at mid-height and gradually decreases to a value of ~0.6°C (~33°F) at H = 100 m (328 ft). Also note that due to two-dimensional effects, the ground temperature below the bottom of the borehole is still affected by heat injection up to a distance of about 50 m (164 ft) below the 100 m (328 ft) long borehole.

As indicated by the dotted lines in *Figure 4*, the average temperatures over the borehole lengths for the three radial distances are 6.2°C, 1.5°C and 0.9°C (43.2°F, 34.7°F and 33.6°F), respectively. This means, for example, that the ground temperature has increased by an average of 0.9°C (1.6°F) (over the borehole length) at a distance of 10 m (32.8 ft) from a borehole after 10 years with a constant heat transfer rate of 6.28 W/m (6.53 Btu/h·ft). These ground temperature increases are directly proportional to the heat transfer rate (see *Equation 1*).

To obtain g-functions, borehole thermal interaction among the three boreholes must be accounted for. A simplified form of the g-function assumes that all heat transfer rates from every borehole are equal. Under this assumption, the borehole wall temperature for the central borehole is influenced by itself and by the other two boreholes located 5 m (16.4 ft) away. Thus, using the principle of spatial superposition and curves for B and r_b on Figure 4, this borehole wall temperature is 9.2°C (= 6.2 + 2 × 1.5) (48.6°F).

This simplified form of the g-function is inadequate for large bore fields.[7] In fact, g-functions are based on a more restrictive assumption that all boreholes have the same borehole wall temperature, T_b. This leads to different heat transfer rates for each borehole. Thus, the borehole wall temperatures of each borehole need to be evaluated using *Equation 3a*.

$$T_{b1} = \left[q_{b1}h_{1,1,r=0.05} + q_{b2}\,h_{2,1,r=5} + q_{b3}\,h_{3,1,r=10}\right]/\,\bar{q}$$
$$T_{b2} = \left[q_{b1}\,h_{1,2,r=5} + q_{b2}h_{2,2,r=0.05} + q_{b3}\,h_{3,2,r=5}\right]/\,\bar{q} \qquad (3a)$$
$$T_{b3} = \left[q_{b1}\,h_{1,3,r=10} + q_{b2}h_{2,3,r=5} + q_{b3}h_{3,3,r=0.05}\right]/\,\bar{q}$$

In these equations, the indices 1 and 3 refer to the outer boreholes, and 2 refers to the middle borehole of a 3 × 1 bore field (*Figure 4*); and $h_{i,j,r=r_i}$ stands for the thermal response factor of borehole i toward borehole j located at a radial distance r_i. For example, with reference to *Figure 4*, $h_{1,2,r=5}$ = 1.5°C (2.7°F). The value of $\bar{q}$ is the average heat transfer rate for the three boreholes, and q_{b1}, q_{b2} and q_{b3} are the heat transfer rates of individual boreholes. The three borehole wall temperatures are equal based on the definition of g-functions. This leads to *Equations 3b*.

$$T_{b1} = T_{b2}$$
$$T_{b2} = T_{b3} \qquad (3b)$$
$$\bar{q} = \left(q_{b1} + q_{b2} + q_{b3}\right)/\,3 = 6.28\ \text{W/m}$$

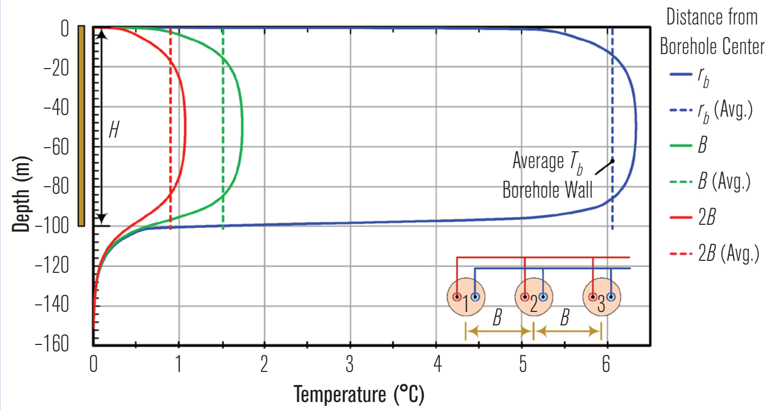
Solving *Equations 3a* and *3b*, the six unknowns are determined: q_{b1} = q_{b3} = 6.58 W/m (6.84 Btu/h·ft), q_{b2} = 5.69 W/m (5.91 Btu/h·ft), T_{b1} = T_{b2} = T_{b3} = 8.66°C (47.6°F) and, using *Equation 1*, the corresponding g-function is 8.66 (intersection of the B / H = 0.05 curve and $\ln(t / t_s)$ = –1.1 in *Figure 2*). As explained earlier, g-functions are independent of T_g and $\bar{q}$, so other values of T_g and $\bar{q}$ would lead to different borehole wall temperatures but the same g-function.

g-functions strongly depend on bore field configuration. GHXSizing, the spreadsheet tool presented in the companion article,[1] provides the capability to evaluate g-functions for regular (with equally spaced boreholes) and irregular configurations. It also includes a section using g-functions to determine the "temperature penalty," T_p, caused by borehole thermal interaction in thermally imbalanced bore fields.[11] This is explained briefly at the end of this article.

Number of Borehole Segments

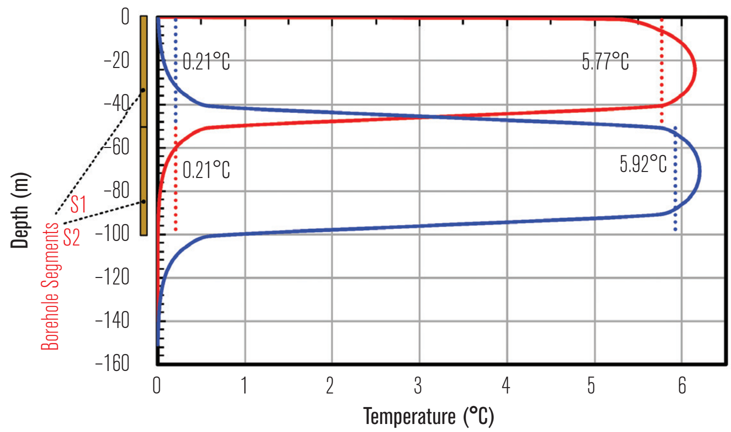
As mentioned earlier, g-functions are based on the assumption that all boreholes have the same borehole

FIGURE 4 Ground temperature increases at three radial distances from a 100 m (328 ft) long borehole after 10 years with q_b = 6.28 W/m (6.53 Btu/h·ft) and T_g = 0°C.



wall temperature, T_b. This is a reasonable assumption considering that all boreholes are typically fed in parallel with the same inlet temperature. There is a further requirement in the g-function definition that T_b should be uniform along the borehole length. *Figure 4* shows that the use of one uniform value of q_b over the entire borehole length does not lead to a uniform T_b. One way to obtain a uniform T_b is to discretize the borehole into several segments with different values of q_b. This is shown in *Figure 5*, where the borehole wall temperature (r_b = 0.05 m [2 in.]) is evaluated for two equal-length borehole segments, S1 (top) and S2 (bottom). The heat transfer rate is assumed uniform along each segment. The two curves represent the T_b profiles resulting from an injection of 6.28 W/m (6.53 Btu/h·ft) in the S1 and S2 segments of the borehole.

FIGURE 5 Temperature profile at r = 0.05 m for a two-segment borehole. The heat transfer rate is 6.28 W/m for each segment.



As indicated in *Figure 5*, the average temperatures of these profiles over S1 and S2 are 5.77°C (42.39°F) and 5.92°C (42.66°F), respectively. The temperature profile resulting from heat injection for S2 is slightly different because the segment is farther away from the surface than S1 is. Also shown in *Figure 5*, the influences of segment S2 on S1 and S1 on S2 are nonnegligible, resulting in an average temperature increase of 0.21°C (0.38°F). When the influences of S1 on S1 and S2 on S1 are superimposed, the resulting average temperature for segment S1 is 5.98°C (42.77°F). Similarly, the resulting average borehole wall temperature for segment S2 is 6.13°C (43.04°F) after superposition. The results show different segment temperatures for the same heat transfer rate.

To follow the g-function assumptions, the segment temperatures are assumed to be identical, giving different heat transfer rates for each segment. Thus, since these increases in temperature are directly proportional to the applied heat transfer rate, the borehole wall temperatures for both segments can be obtained using *Equation 4a*.

$$\begin{aligned} T_{b1} &= \left[q_{b1} h_{1,1,r=0.05} + q_{b2} h_{2,1,r=0.05} \right] / \bar{q} \\ T_{b2} &= \left[q_{b1} h_{1,2,r=0.05} + q_{b2} h_{2,2,r=0.05} \right] / \bar{q} \end{aligned} \tag{4a}$$

where indices 1 and 2 refer here to S1 and S2, respectively. Because the borehole wall temperature over the entire length has to be uniform, the average heat transfer rate is given by the average of both heat transfer rates:

$$\begin{aligned} T_{b1} &= T_{b2} \\ \bar{q} &= \left(q_{b1} + q_{b2} \right) / 2 = 6.28 \text{ W/m} \end{aligned} \tag{4b}$$

Solving *Equations 4a* and *4b* leads to q_{b1} = 6.37 W/m (6.63 Btu/h·ft), q_{b2} = 6.19 W/m (6.44 Btu/h·ft) and T_{b1} = T_{b2} = 6.06°C (42.91°F). In other words, to maintain a uniform borehole wall temperature of 6.06°C (42.91°F), heat transfer rates of 6.37 W/m and 6.19 W/m (6.63 Btu/h·ft and 6.44 Btu/h·ft) are required in segments S1 and S2, respectively. The g-function for this single borehole calculated with two segments is then 6.06, compared to a g-function of 6.2 calculated for the same borehole with one segment (*Figure 4*). Thus, segmentation of the borehole influences the calculated value of the g-function. In this example, only one borehole is considered. If the 3 × 1 geometry used earlier were calculated with two segments, *Equations 4a* and *4b* would contain six different wall temperatures and heat transfer rates.

It is not hard to imagine that the problem becomes computationally intensive when the number of boreholes and segments per borehole is large. The complexity is further increased by the fact that when the bore field is sized, the length is unknown a priori, and, as shown in the companion article,[1] g-functions need to be calculated at each iteration.

Figure 6 shows the g-functions of a 5 × 5 borehole field, evaluated with 1 and 12 segments using the input parameters reported in *Table 1*. It can be seen that g-function values evaluated with 12 segments are lower than those for one segment, especially for large values of $\ln(t/t_s)$, where the difference can reach more than 6%. It should be noted, however, that for typical boreholes, g-function values for $\ln(t/t_s)>2$ are seldom reached in practice because they correspond to very long operating times (*Figure 3*). An example given in the companion article[1] shows a reduction of 2.6% in the required borehole length for typical conditions when 12 segments are used instead of 1. It can be concluded that bore field sizing based on g-functions evaluated with a large number of segments will lead to slightly shorter boreholes. In other words, bore field sizing using g-functions generated with fewer segments is more conservative.

Finally, if steady-state heat transfer is assumed in the borehole, then the mean fluid temperature, T_m, can be obtained using a steady-state borehole thermal resistance, R_b:

$$T_m = T_b + q_b R_b = q_b\left(\frac{g}{2\pi k_g} + R_b\right) + T_g \quad (5)$$

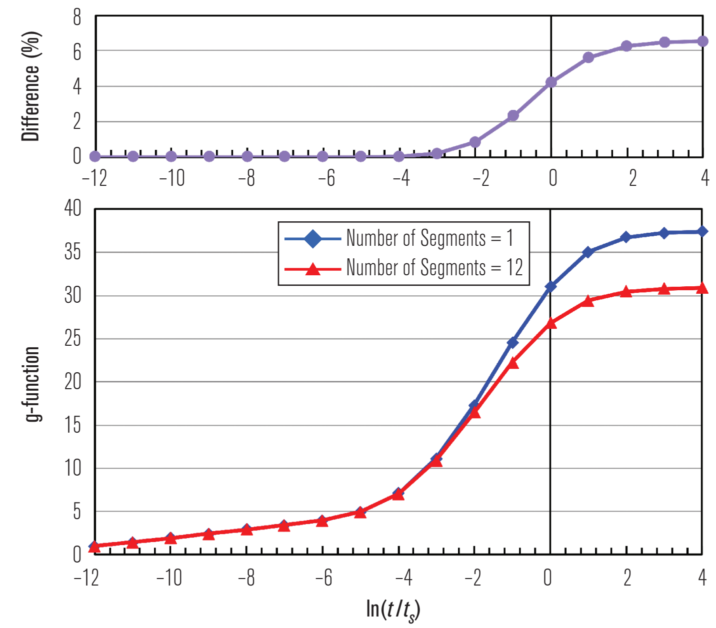
Values of R_b are tabulated in the *ASHRAE Handbook*[12] or can be evaluated using the GHXSizing spreadsheet discussed in the companion article.[1] They typically range from 0.05 m·K/W to 0.2 m·K/W (0.09 h·ft·°F/Btu to 0.34 h·ft·°F/Btu). The value of R_b has a significant impact on sizing; small values of R_b lead to shorter boreholes.

Equation 5 is based on "long-term" g-functions, which neglect the unsteady short-term effects linked to borehole thermal capacity. There are several ways to account for unsteady heat transfer in the borehole from the mean fluid temperature to the borehold wall temperature (i.e., from T_m to T_b in *Figure 1*). One of these ways is through what are called short-term g-functions because they are a natural extension of g-functions. The reader is referred to the works of Yavuzturk and Spitler,[3] Xu and Spitler,[4] and Brussieux and Bernier[5] for more details on short-term g-functions.

Temperature Penalty

When the classic ASHRAE sizing equation is used, the ground temperature needs to be corrected with a temperature penalty, T_p, to account for borehole thermal interaction. In GHXSizing, the temperature penalty can be evaluated using g-functions.[11]

FIGURE 6 g-function curves of 5 × 5 bore field evaluated with 1 and 12 segments.



The temperature penalty for a field of N_b boreholes is evaluated based on the following equation:

$$T_p = \frac{q_y\left(g_{N_b}(t) - g_1(t)\right)}{2\pi k_g} \quad (6)$$

where q_y is the yearly annual ground load divided by the boreholes' overall length ($N_b \times H$), g_{N_b} and g_1 are, respectively, the g-functions of the entire bore field and of one single borehole, and t is the design period.

As an example, GHXSizing determines a T_p value of 2.6°C (4.7°F) for a 3 × 1 bore field with geometry A (*Table 1*), a yearly annual ground thermal imbalance of 6.28 W/m and a design period of 10 years. This means that the borehole wall temperatures of the 3 × 1 bore field increases by 2.6°C (4.7°F) more than the borehole wall temperature of one single borehole after 10 years of operation. This value can also be estimated using *Figure 2* by subtracting the g-function (g_1= 6.1) for the single borehole ($B/H = \infty$) at $\ln(t/t_s) = -1.1$ from the g-function (g_3 = 8.7) for $B/H = 0.05$.

Concluding Remarks

This article reviews how thermal response factors, also known as g-functions, are calculated from nondimensional parameters and used for sizing VHGEs.

g-functions are used to obtain the borehole wall temperature for a given heat transfer rate, ground thermal conductivity and ground temperature (*Equation 1*). g-functions can be calculated using the FLS model, an analytical solution to 2D heat transfer around the borehole. Then, spatial superposition can be used to account for borehole-to-borehole thermal interaction. Discretization of the borehole into a number of segments is shown to have an influence, albeit small for typical conditions, on the value of the g-function. The evaluation of temperature penalties based on g-functions is also presented. In closing, it is hoped that g-functions will be better understood by design engineers and that the concepts and examples provided here will find their way into the *ASHRAE Handbook*.

Supplementary Material

The spreadsheet tool called GHXSizing mentioned in this article can be found on ASHRAE's website at https://tinyurl.com/mbmdpbv4 and on the author's website (https://tinyurl.com/GeoSizing)

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